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*** Dr. G. B. M. Zerr and Mr. J. F. Lawrence prove in general that $a_1 + a_2 + \dots + a_n > n\sqrt[n]{(a_1 a_2 \dots a_n)}$. Dr. Zerr also proved that $(a_1^m + \dots + a_n^m)/n > [(a_1 + \dots + a_n)/n]^m$.

210. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

The sum of five quantities and the sum of their cubes are both zero. Show that the sum of their fifth powers is a factor of the sum of any odd powers of the quantities.

Solution by G. W. GREENWOOD, M. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

Denote the quantities by $\alpha, \beta, \gamma, \delta, \varepsilon$, and let the equation of which they are the roots be

$$\alpha^5 + a\alpha^4 + b\alpha^3 + c\alpha^2 + d\alpha + e = 0.$$

Then

$$\sum \alpha = -a = 0.$$

$$\sum \alpha^3 = (\sum \alpha^2 - \sum \alpha \beta) \sum \alpha - \sum \alpha \beta \gamma = c = 0.$$

Substituting the roots in turn for α , and adding, we get,

$$\sum \alpha^5 + 5e = 0.$$

Multiply the equation by α^2 , make the same substitutions, and we get in a similar manner,

$$\sum \alpha^7 + b \sum \alpha^5 + 5e = 0,$$

$$\text{i. e., } \sum \alpha^7 + (b-1) \sum \alpha^5 = 0.$$

Hence $\sum \alpha^5$ is a factor of $\sum \alpha^7$ and the process may be repeated indefinitely.

Also solved by Elmer Schuyler, G. B. M. Zerr, J. Scheffer.

GEOMETRY.

228 Proposed by O. E. GLENN, A. M., Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa

Given a point O without a circle S ; two arbitrary lines through O cut S in the points A, A' , and B, B' , respectively. Prove, by pure geometry, that the four circles through $OAR, OBR, OA'R', OB'R'$, respectively, intersect in points collinear with O ; R and R' being points upon S arbitrarily chosen.

Solution by T. L. CROYES, Paris, France.

Let us take the inverse of the system with regard to O , and let the inverses of the five circles $S, OAR, OBR, OA'R', OB'R'$, be a circle s , and the four right lines $ar, br, a'r', b'r'$ (a, a', b, b', r, r' being the inverses of A, A', B, B', R, R').

By Pascal's theorem, the points of intersection $(ar, b'r')(br, a'r')$ are collinear with the point O .

237. Proposed by S. A. COREY, Hiteiman, Iowa.

Let AB, BC, CD, DE, EA be the sides of a pentagon, plain or gauche. Double the length of CB and DE by extending from B and E to G and H , re-

spectively. Draw $B'D$ parallel to and of the same currency as BC . Connect G and H . Then prove that $2(AB^2 + BO^2 + OD^2 + DE^2 + EA^2) = 3OD^2 + 4(DE \cdot BC \cdot \cos EDB' + EA \cdot AB \cdot \cos EAB) + GH^2$.

Solution by the PROPOSER.

Let a, b, c , and d be the vector sides corresponding with DE, EA, AB , and BC , respectively. Then will $-(a+b+c+d)$ represent the side CD , and $(a-b-c+d)$, the line GH . Squaring each of these six vectors to get their squared tensors, and adding as follows:

$$\begin{aligned} & 2[T^2a + T^2b + T^2c + T^2d + T^2 \cdot (-a-b-c-d)] \\ & - 3T^2 \cdot (-a-b-c-d) - T^2 \cdot (a-b-c+d) \end{aligned}$$

is found to be $4(Ta \cdot Td \cdot \cos ad + Tb \cdot Tc \cdot \cos bc)$ as required by the problem.*

238. Proposed by O. W. ANTHONY, Head of the Mathematical Department, DeWitt Clinton High School, New York.

Construct a trapezoid having given the sum of the parallel sides, the sum of the diagonals, and the angle formed by the diagonals.

I. Solution by J. H. MEYERS, S. J., Professor of Mathematics in Spring Hill College, Mobile, Ala.

Construct the triangle ABC , whose base AB =sum of parallel sides, angle C =angle between diagonals and where $AC+CB$ =sum of diagonals. [See Casey's *Sequel to Euclid*, Book III, Prop. 29.]†

Take the point E on AB , such that $CE=CB$; from E draw EF parallel and equal to AC meeting CB in O ; join B and F . $CFBE$ is the required trapezoid. In proof, $AE=CF$, hence $EB+CF$ =given sum of parallel sides. Since $EF=AC$, $EF+CB$ =given sum of parallel sides. And finally, angle $EOB=ACB$.

Analogously solved by J. Scheffer.

II. Remark by G. W. GREENWOOD, M. A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

Consider any trapezoid $ABCD$ satisfying the given conditions, where AB, CD are the parallel sides. Draw any parallel to AC meeting AB internally in A' , say, and hence meeting CD internally in C' . The trapezoid $A'BC'D$ can easily be seen to fulfill the required conditions. Therefore the solution is not unique.

*In the above solution the lines CB and DE are the ones produced. Prof. G. W. Greenwood has demonstrated that the corresponding theorem, where the lines produced are BC and ED , does not hold true. Ed. E.

†Describe on the same side of AB as a chord the arcs AXB, AYB such that angle AXB =given angle α for every point X , and $AYB=\frac{1}{2}\alpha$. Determine the point P on arc AYB such that AP =sum of diagonals of trapezoid (i. e., sum of sides of the required triangle). Let AP meet arc AXB in C . Then since angle $OPB+CPB=\alpha$, angle $CPB=\frac{1}{2}\alpha$ and $CP=CB$. Therefore ABC is the required triangle. Ed. E.